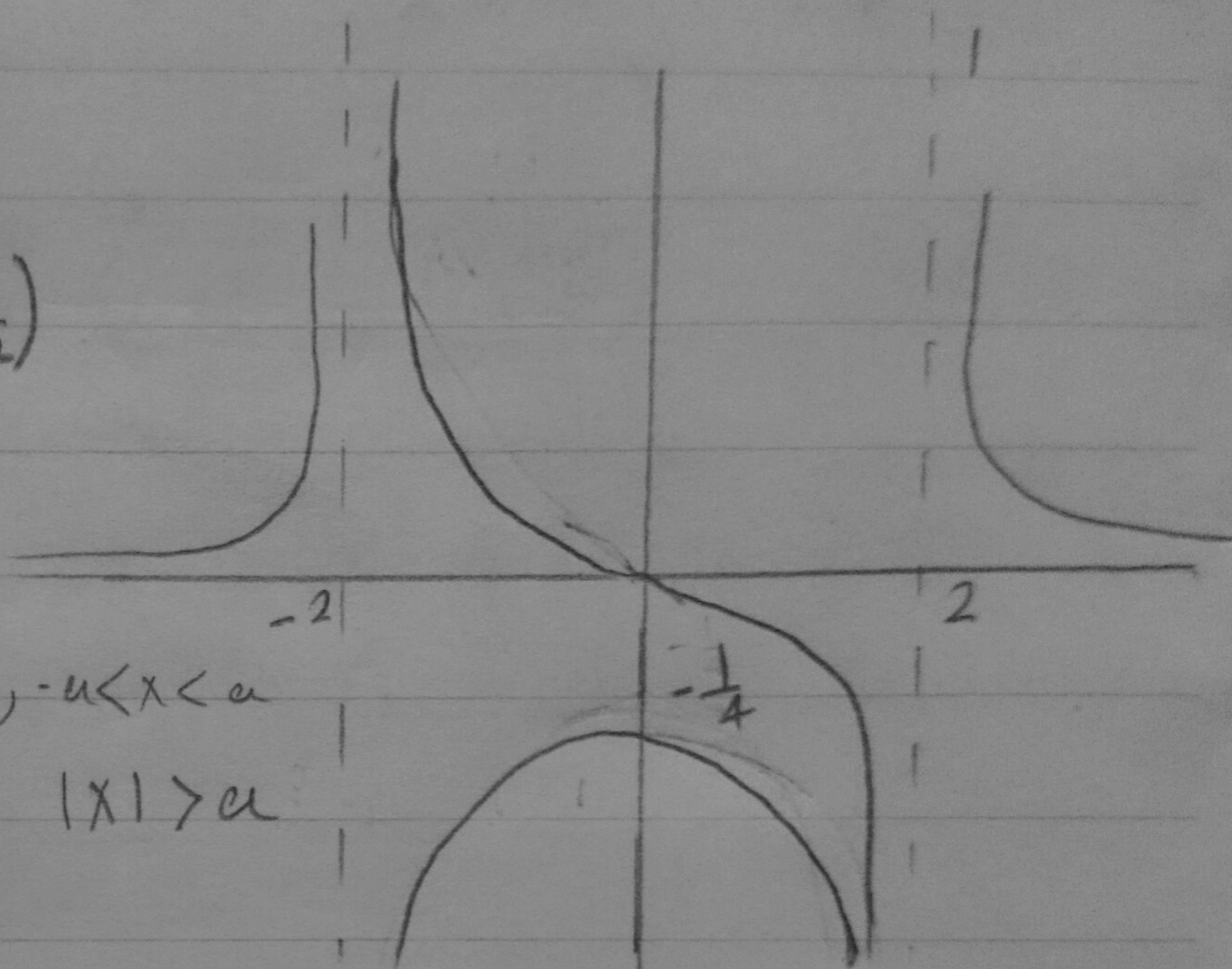


Integration by using Partial fractions

Remember

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{a^2 - x^2} dx = \begin{cases} \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right), & -a < x < a \\ -\frac{1}{a} \coth^{-1}\left(\frac{x}{a}\right), & |x| > a \end{cases}$$



$$\int \frac{u}{u} dx = \ln|u|$$

$$\frac{P(x)}{Q(x)} = x^3 - x + 4$$

$$= P_x + \frac{R(x)}{Q(x)}$$

$$y = \frac{-1}{2^2 - x^2}$$

Find $\int \frac{1}{a^2 - x^2} dx$

$$F(x) = \frac{1}{a^2 - x^2} = \frac{1}{(a-x)(a+x)} = \frac{A}{a-x} + \frac{B}{a+x}$$

$$\frac{1}{a^2 - x^2} = \frac{A(a+x) + B(a-x)}{(a-x)(a+x)}$$

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$$Aa + Ax + Ba - Bx = 1$$

$$a(A+B) + x(A-B) = 1$$

$$A - B = 0 \quad (1)$$

$$a(A+B) = 1$$

$$A + B = \frac{1}{a} \quad (2)$$

$$2A = \frac{1}{a}$$

$$A = \frac{1}{2a}, \quad B = \frac{1}{2a}$$

$$\frac{1}{a^2 - x^2} = \frac{-1}{2a} \left[\frac{-1}{a-x} \right] + \frac{1}{2a} \left[\frac{1}{a+x} \right]$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{-1}{2a} \ln(a-x) + \frac{1}{2a} \ln(a+x)$$

$$= \frac{1}{2a} \left[-\ln(a-x) + \ln(a+x) \right] = \frac{1}{2a} \ln \left(\frac{a+x}{a-x} \right) \quad \begin{matrix} x \neq a \\ a \neq 0 \end{matrix}$$

$$\star \int \frac{dx}{x^2 - 4} = - \int \frac{dx}{4 - x^2} = - \int \frac{dx}{2^2 - x^2} = -\frac{1}{4} \ln \frac{2+x}{2-x} \quad \#$$

EX 2.

Pg 24

$$\int \frac{2x}{x^3+4x}$$

$$\frac{1}{x^3+4x} = \frac{1}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$\frac{a}{b} = \frac{a}{b}$$

$$A(x^2+4) + x(Bx+C) = 1$$

$$Ax^2 + 4A + Bx^2 + Cx = 1$$

$$x^2(A+B) + Cx + 4A = 1$$

Comparing The Coefficients

$$A+B=0$$

$$C=0$$

$$1=4A$$

$$A = \frac{1}{4}, B = -\frac{1}{4}, C = 0$$

$$\text{Then } I = \int \frac{1}{x^3+4x} = \frac{1}{4} \int \frac{1}{x} dx - \frac{1}{4 \times 2} \int \frac{2x}{x^2+4}$$

$$I = \frac{1}{4} \ln x - \frac{1}{8} \ln(x^2+4) \quad \#$$

$$\text{Ex 6: (18)} \quad \int \frac{x^5 - 2}{x^3 + x^2} dx$$

$$= x^2 - x + 1 - \frac{x^2 + 2}{x^2(x+1)}$$

$$\frac{1}{x+1} + \frac{2}{x^2(x+1)}$$

$$\frac{2}{x^2(x+1)} = \frac{A}{x^2} + \frac{B}{x+1} + \frac{C}{x}$$

$$\frac{2}{x^2(x+1)} = \frac{A(x+1) + Bx^2 + C(x+1)x}{x^2(x+1)}$$

$$\begin{cases} 2 = A(0+1) \\ A = 2 \end{cases} \quad \begin{cases} 2 = B(-1)^2 \\ B = 2 \end{cases}$$

$$x=1 \quad 2 = A(2) + 2(1)^2 + 2C \cdot 1$$

$$2 = 2(2) + 2(1) + 2C$$

$$2C + 4 = 0$$

$$C = -2$$

$$\int \left(x^2 - x + 1 - \frac{3}{x+1} - \frac{2}{x^2} + \frac{2}{x} \right) dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - 3 \ln(x+1) + 2 \ln x + \frac{2}{x} + C$$

$$\begin{array}{r} x^2 - x + 1 \\ x^3 + x^2 \overline{) x^5 - 2} \\ \underline{\ominus x^5 \oplus x^4} \\ x^4 \end{array}$$

$$\begin{array}{r} -x^4 - 2 \\ \oplus x^4 \ominus x^3 \\ \hline -x^3 - 2 \end{array}$$

$$\begin{array}{r} x^3 - 2 \\ \ominus x^3 \oplus x^2 \\ \hline -x^2 - 2 \end{array}$$

$$-x^2 - 2$$

$$Ex: \int \frac{dx}{x^2 + ax + a^2}$$

$$x^2 + 5x + 4 = (x+1)(x+4)$$

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$$\begin{aligned} x^2 + 3x + 4 &= x^2 + 3x + \left(\frac{3}{2}\right)^2 + 4 - \left(\frac{3}{2}\right)^2 \\ &= \left(x + \frac{3}{2}\right)^2 + 4 - \frac{9}{4} \\ &= \left[x + \frac{3}{2}\right]^2 + \frac{7}{4} \\ &= \left[x + \frac{3}{2}\right]^2 + \left(\frac{\sqrt{7}}{2}\right)^2 \end{aligned}$$

$$\begin{aligned} x^2 + ax + a^2 &= x^2 + 2\frac{a}{2}x + \left(\frac{a}{2}\right)^2 + a^2 - \left(\frac{a}{2}\right)^2 \\ &= \left(x + \frac{a}{2}\right)^2 + \frac{3a^2}{4} \\ &= \left(x + \frac{a}{2}\right)^2 + \left(\frac{a\sqrt{3}}{2}\right)^2 \end{aligned}$$

$$\int \frac{dx}{x^2 + ax + a^2} = \int \frac{1}{\left(x + \frac{a}{2}\right)^2 + \left(\frac{a\sqrt{3}}{2}\right)^2} dx$$

$$\int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

$$\text{Let } x + \frac{a}{2} = u$$

$$d\left[x + \frac{a}{2}\right] = du$$

$$dx = du$$

$$I = \int \frac{1}{u^2 + \left(\frac{\sqrt{3}a}{2}\right)^2}$$

$$= \frac{1}{\frac{\sqrt{3}a}{2}} \operatorname{Tan}^{-1} \frac{X + \frac{a}{2}}{\frac{\sqrt{3}a}{2}}$$

$$= \frac{2}{\sqrt{3}a} \operatorname{Tan}^{-1} \frac{2(X + \frac{a}{2})}{\sqrt{3}a} \quad \#$$

E. X: 2 Pg 24: $\int \frac{X^3}{X^3 - a^3} dX$

$$\frac{X^3}{X^3 - a^3} = 1 + \frac{a^3}{X^3 - a^3}$$

$$\frac{a^3}{X^3 - a^3} = \frac{a^3}{(X - a)(X^2 + aX + a^2)}$$

$$= \frac{A}{(X - a)} + \frac{BX + C}{X^2 + aX + a^2}$$

$$a^3 = A(X^2 + aX + a^2) + (BX + C)(X - a)$$

$$X = a$$

$$a^3 = A(a^2 + a^2 + a^2) + 0$$

$$3a^2 A = a^3 \Rightarrow A = \frac{a}{3}$$

$$X = 0$$

$$a^3 = a^2 A + (0 + C)(0 - a)$$

$$a^3 = a^2 \cdot \frac{a}{3} - aC$$

$$a^3 = \frac{a^3}{3} - aC \Rightarrow C = \frac{-2a^2}{3}$$

$$\text{Let } X = 1$$

$$a^3 = A(1+a+a^2) + (B+C)(1-a)$$

$$a^3 = \frac{a}{3}(1+a+a^2) + \left(B - \frac{2}{3a^2}\right)(1-a)$$

$$B = -\frac{1}{3}a$$

$$\frac{a/3}{X-a} + \frac{-\frac{a}{3}X - \frac{2a^2}{3}}{X^2+ax+a^2}$$

$$I = \int \left(\frac{-\frac{a}{3}X}{X^2+ax+a^2} - \frac{\frac{2a^2}{3}}{X^2+ax+a^2} \right) dx$$

$$I = -\frac{a}{3} \int \frac{X dx}{X^2+ax+a^2} - \frac{2a^2}{3} \int \frac{dx}{X^2+ax+a^2}$$

$$J = \frac{X dx}{X^2+ax+a^2}$$

$$X^2+ax+a^2 \rightarrow 2X+a$$

$$= \frac{(2X+a-a)/2 dx}{X^2+ax+a^2}$$

$$= \frac{1}{2} \int \frac{2X+a}{X^2+ax+a^2} dx - \frac{a}{2} \int \frac{dx}{X^2+ax+a^2}$$

$$= \frac{1}{2} \ln(X^2+ax+a^2) - \frac{a}{2}$$